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# Assessment of diffusion coefficients of general solvents by PFG-NMR: Investigation of the sources error

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# Abstract

Accurate measurements of the diffusion coefficients, including an estimate of uncertainty, of various solvent molecules using the PFG-NMR method were performed in this study. Accurate diffusion coefficients were obtained using the Shigemi NMR tube. The relative combined standard uncertainties of the diffusion coefficients were found to be within approximately 0.4%. The three uncertainty sources (signal decay of the standard and the solvent, and diffusion coefficient of standard) equally affect the combined standard uncertainties. Unreliable data were obtained using a normal NMR tube, indicating that convection and background gradient effects significantly affected the accurate measurement of the diffusion coefficients.

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### 1. Introduction

A powerful tool is required for the detection and identification of diffusion in a solution on each chemical and biological field, for example: evaluation of encapsulation [1–7]; highlighting the chemical exchange reaction [8–12]; and the estimation of size for supramolecular and dendritic materials [13–17]. Since the diffusion coefficient is important for evaluating the physicochemical properties of a molecule, it is necessary to determine the coefficient accurately. For instance, one needs to use a precise diffusion coefficient to calculate the rate constant of the photochemical bimolecular reaction using a Stern–Volmer type analysis. Moreover, such values are indispensable for the accurate evaluations in transport studies, rheology, and fluid chemistry in both experimental and computational fields. The dynamic light scattering (DLS) approach is known to detect diffusion phenomena of molecules on the 1 nm scale. On the other hand, the pulsed field gradient nuclear magnetic resonance (PFG-NMR) spectroscopic method has recently achieved a developmental stage for the quantitative measurement of self-diffusion coefficients. This method exhibits the potential to provide a mathematical, and physical method of prediction of diffusion phenomena for much smaller targets in solution than DLS. In addition, it is possible to distinguish the individual diffusion components by monitoring NMR signals at different chemical shifts simultaneously [18–20].

In this report, we examined the sources and magnitude of errors in the PFG-NMR method. At first the international guide for the estimation of the uncertainty was employed, often called GUM, published by the standardization bodies, such as the Bureau International des Poids et Mesures (BIPM) and the International Organization for Standardization (ISO). The difficulty in the estimation of the diffusion coefficient, measured using the PFG-NMR, arises from many uncertainty factors including components

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of minor errors. Furthermore, for polymeric sized compounds, one has to consider not only basic parameters but also complicated effects, such as homonuclear *J*-evolution effects and polydispersity. Therefore, first of all, the small molecules (methanol, ethanol, benzene, dichloromethane, chloroform, hexane, cyclohexane, and acetonitrile), which are often used as a solvent, were targeted. We investigated the accuracy of diffusion coefficients observed using the PFG-NMR under limited conditions.

In the past a significant number of the published diffusion coefficients for solvent molecules were obtained using the diaphragm and capillary-cell protocols [21-39], and PFG-NMR [40-47]. However, the published values exhibit scatter and are not comparable because the first method requires tracer isotope molecules. One has to consider the mass effect which introduces a significant error in the estimation of the diffusion coefficient [21,48]. Even though the PFG-NMR method can be used to perform direct measurements of the diffusion coefficient without the mass effect of the tracer isotope, there are considerable convection effects when one performs the measurement using non-viscous solvents which exhibit a low boiling point and heat capacity. To remove such effects, Holz carried out careful measurements of organic solvents using a special temperature controlling system [42,43]. In another case, to compensate for the convection effect on coherence, a selection using gradients (a specially designed pulse sequence) was performed by Keeler and Müller [49]. In addition, Jokisaari used a spinning method, rotating the sample tube at a rotation speed of the order of 10 Hz, to eliminate convection effects [50].

In the high-resolution NMR experiment it is common to perform experiments without spinning samples for the purpose of minimizing possible vibration instabilities, spinning side bands and background gradients to make accurate quantitative observations. Thus, in order to prevent these effects using normal NMR, we used a Shigemi symmetrical NMR tube which acts as a thermal insulating system and increases both accuracy and reproducibility of the diffusion measurements by reducing the chance of convection in the sample. It consisted of the same or similar magnetic susceptibility matched glass to solvents [51]. Furthermore, we limited the height of the sample to 2 mm for all measurements to reduce the convection effect. One can imagine that decreasing the sample height leads to a low signal-to-noise ratio and boundary-induced field inhomogeneity problems due to the reduction of the sample volume. However, such magnetic susceptibility controlled glass makes it possible to avoid such effects.

# 2. Experimental

NMR measurements were performed on a Varian  $^{\text{UNITY}}$ INOVA 600A (14.1 T) spectrometer equipped with a H-F{X} diffusion probe (DSI-V218, Doty Scientific) capable of producing magnetic field pulse gradients in the *z*-direction of approximately 2528 G cm<sup>-1</sup>. NMR lock

was not used for all experiments and the temperature was set at 298.15 K ( $\pm 0.1$ ). Temperature calibration on the spectrometer was performed using ethyleneglycol as a standard. All experiments were carried out using 5 mm o.d. Shigemi microcell NMR tubes (BMS-005V, CMS-005V, MMS-005V, and DMS-005V, Shigemi). The sample height was 2 mm. The diffusion coefficients were calculated using the Stejskal-Tanner diffusion equation [36], i.e.,  $\ln(I/$  $I_0 = -D\gamma^2 G^2 \delta^2 (\Delta - \delta/3)$  only for data where the correlation coefficient of  $\ln(I/I_0)$  versus  $\gamma^2 G^2 \delta^2 (\Delta - \delta/3)$  was higher than 0.99. To determine the reliable size of the random error of the spectral repetitions, a single spectrum of the solvent was chosen as a target; a more efficient way to observe the reliability of quantitative results. The individual peak width was between 6 and 10 Hz with an assumed lognormal probability distribution. A signal-to-point-topoint noise ratio between 700 and 1200 was employed.

The PFG spin echo (PFGSE) sequence  $[(\pi/2)-\tau-(\pi)-\tau$ acquisition] [52,53] was used (see Fig. 1A) in this work. The PFG stimulated echo (PFGSTE) sequence  $[(\pi/2)-\tau_2-(\pi/2)-\tau_1-(\pi/2)-\tau_2-$ acquisition] [54] is described in Fig. 1B. Rectangular pulsed gradients of 1 ms duration were incremented from 0 to 70 G cm<sup>-1</sup> in 10 steps with 4 averaged transients. The  $\pi/2$  pulse widths of 12.90 µs, 100 s relaxation delay, and 3.5 s acquisition time were predefined. The interval between the gradient pulses ( $\Delta$ ) was set to 50 ms.

Water was purified by micro porous filtration using a water purification system manufactured in Millipore S.A. Methanol (99.7 atom%), benzene (99.0 atom%), ethanol (99.0 atom%), cyclohexane (99.0 atom %), hexane (99.0 atom%), acetonitrile (99.0 atom%), dichloromethane (99.0 atom%), and chloroform (99.6 atom%) were purchased from Wako Pure Chemical Industries.

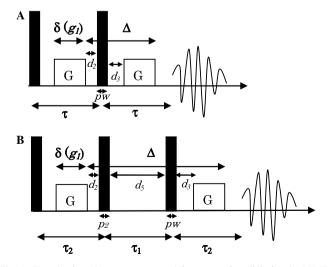


Fig. 1. Two basic pulse sequences used for measuring diffusion by NMR: (A) PFGSE; (B) PFGSTE.

# 3. Calculational method of the diffusion coefficient by PFG-NMR

Using common Stejscal–Tanner pulse sequences, the diffusion is characterized by the attenuation of the signal in the NMR experiment. The attenuation of the PFGSE and PFGSTE methods in a time-independent gradient are given, respectively, by:

$$\ln(I/I_0)_i = -2\tau/T_2 - AG_i^2,\tag{1}$$

$$\ln(I/I_0)_i = -\tau_1/T_1 - 2\tau_2/T_2 - \ln 2 - AG_i^2,$$
(2)

and

$$A = -\alpha D\gamma^2 \delta^2 (\Delta - \delta/3), \tag{3}$$

where  $I/I_0$  is the ratio between observed intensities (I) and the intensities for a single pulse  $(I_0)$ ,  $\tau_1$  is the time between the second and third pulses,  $\tau_2$  ( $\tau$ ) is the time between the first two pulses,  $T_1$  is the longitudinal relaxation time,  $T_2$ is the transverse relaxation time, D is the diffusion coefficient without standardization,  $\gamma$  is the gyromagnetic ratio,  $\delta$  is the gradient pulse length,  $\Delta$  is the time between the start of the first gradient pulse and the start of the second, *i* is the sampling number, G is the gradient strength, and  $\alpha$ is standardization parameter (this parameter is optimized by water as follows method). The negative slope in the plot of the logarithmic signal vs.  $G^2$  represents the diffusion coefficient D as a consequence of Gaussian propagation using Eq. (3).

$$\ln(I_{\rm H_{2}O}/I_{0,\rm H_{2}O})_{i} = -2\tau_{\rm H_{2}O}/T_{2,\rm H_{2}O} - A_{\rm H_{2}O}G_{i,\rm H_{2}O}^{2},$$
(4)

$$A_{\rm H_{2O}} = -\alpha D_{\rm H_{2O}} \gamma^2 \delta_{\rm H_{2O}}^2 (\Delta_{\rm H_{2O}} - \delta_{\rm H_{2O}}/3), \tag{5}$$

$$D_{\text{sample}} = \frac{A_{\text{sample}} D_{\text{H}_{2}\text{O},\text{ref}} \delta_{\text{H}_{2}\text{O}}^{2} (\Delta_{\text{H}_{2}\text{O}} - \delta_{\text{H}_{2}\text{O}})/3}{\delta_{\text{sample}}^{2} (\Delta_{\text{sample}} - \delta_{\text{sample}}/3) A_{\text{H}_{2}\text{O}}},$$
(6)

where  $D_{\text{sample}}$  is the true diffusion coefficient of the solvent after correction,  $D_{\text{H}_2\text{O}}$  is the diffusion coefficient of water without correction,  $D_{\text{H}_2\text{O},\text{ref}}$  is the diffusion coefficient of water in the literature (2.299 × 10<sup>-9</sup> m<sup>2</sup> s<sup>-1</sup>) and  $A_{\text{sample}}$  is the slope of the sample in Eqs. (1) or (2). As for the calibration for  $\alpha$ , initially one must measure the diffusion coefficient of water without standardization using Eq. (1) (see Eqs. (4) and (5), where the subscript denotes sample name). Using the observed  $D_{\text{H}2\text{O}}$ , one can obtain the corrected, or true diffusion coefficient of the solvents ( $D_{\text{sample}}$ ) using Eq. (6).

$$D_{\text{sample}} = A_{\text{sample}} \frac{D_{\text{H}_2\text{O},\text{ref}}}{A_{\text{H}_2\text{O}}}.$$
(7)

If one uses the same value of  $\Delta$  and  $\delta$  in both measurements of water and the sample, Eq. (4) reduces to Eq. (7), where  $A_{\rm H_2O}$  is the slope of the water in Eqs. (1) or (2).

In this work, the examination of all of the above parameters for the PFGSE method was performed.

### 4. Concept (identifying and analyzing uncertainty sources)

#### 4.1. Gradient pulse length $(\delta)$

Since  $\delta$  is equal to  $g_1$ , which is the gradient pulse length, the experimental standard deviation of  $\delta$  observed for repeated measurements is given by

$$\sigma^2(\delta) = \sigma^2(g_1). \tag{8}$$

The standard uncertainly of  $\delta$  defined according to the GUM is the same as the standard deviation of  $\delta$ . The estimated value of the time variation was found to be 12.5 ns for the NMR instrument. Thus the standard deviation of  $\delta$  was calculated using the assumption of a rectangular distribution for that variation, i.e.,  $12.5 \text{ ns}/\sqrt{3} = 7.22 \text{ ns}.$ 

## 4.2. Interval between gradient pulses $(\Delta)$

In the PFGSE method,  $\Delta$  is the sum of  $g_1, d_2, pw$ , and  $d_3$ . On the other hand, as with the PFGSTE method,  $\Delta$  is comprised of  $g_1, d_2, pw, p_2, d_5$ , and  $d_3$ , where  $g_1$  is equal to  $\delta, d_2$  is the time between the end of the first gradient and the second pulse, pw is the second (PFGSE) and the third (PFG-STE) pulse length,  $p_2$  is the second pulse length for PFGSTE method,  $d_5$  is the time between the second and the third pulse, and  $d_3$  is the time between the last pulse and the second gradient.

$$\sigma^{2}(\Delta) = \sigma^{2}(g_{1}) + \sigma^{2}(d_{2}) + \sigma^{2}(pw) + \sigma^{2}(d_{3}),$$
(9a)  
$$\sigma^{2}(\Delta) = \sigma^{2}(g_{1}) + \sigma^{2}(d_{2}) + \sigma^{2}(p_{2}) + \sigma^{2}(d_{5}) + \sigma^{2}(pw) + \sigma^{2}(d_{3}).$$

The standard deviation of  $\Delta$ , or standard uncertainly, is represented by Eq. (9a) for PFGSE and Eq. (9b) for PFGSTE.

The time variations are 25.0 and 30.6 ns by Eqs. 9a and 9b, respectively. Therefore, the respective standard deviations are 14.4 and 17.6 ns, due to the rectangular distribution assumption.

# 4.3. Slope of water attenuation $(A_{\rm H_2O})$

The diffusion coefficient of water  $D_{H_2O}$  is calculated by

$$\ln(I/I_0)_{i,\mathrm{H}_2\mathrm{O}} = -2\tau/T_{2,\mathrm{H}_2\mathrm{O}} - A_{\mathrm{H}_2\mathrm{O}}G_i^2.$$
 (10)

Using the linear least-squares fitting procedures in the PFGSE method, one can first calculate  $A_{\rm H_2O}$  as the slope of the fitting line of the plot of experimental values of  $\ln(I/I_0)_{i,\rm H_2O}$  as a function of  $G_i$ . The uncertainty of  $A_{\rm H_2O}$  arises from the deviation of the experimental data points from the linear least-squares line.

$$\sigma(A_{\rm H_2O}) = \frac{s_{y/x}(A_{\rm H_2O})}{\left\{\sum_i \left(G_i^2 - \overline{G_i^2}\right)^2\right\}^{0.5}},$$
(11)

where

$$s_{y/x}(A_{\rm H_2O}) \left\{ \frac{\sum_{J=1}^{n} \left[ \ln(I/I_0)_{\rm H_2O} - \ln(I/\hat{I_0})_{\rm H_2O} \right]^2}{n-2} \right\}^{0.5}.$$
 (12)

The uncertainty of  $A_{\rm H_2O}$  is calculated by Eqs. (11) and (12).

# 4.4. Diffusion coefficient of water $(D_{H_{2}O,ref})$

In this work we used H<sub>2</sub>O ( $D_{\rm H_2O,ref}$  : 2.299×  $10^{-9}$  m<sup>2</sup> s<sup>-1</sup>) as a standard. Mill estimated the uncertainty of the value for water ( $\sigma(D_{\rm H_2O,ref})$ ) as 0.3% [21]. Therefore, we used this value as the standard deviation of  $D_{\rm H_2O,ref}$ . Since there is no additional information about the uncertainty value, a rectangular distribution is assumed and the standard deviation or standard uncertainly is defined as  $0.398 \times 10^{-11}$ .

# 4.5. Slope of sample attenuation $(A_{sample})$

The standard deviation or standard uncertainty of  $A_{\text{sample}}$ ;  $\sigma(A_{\text{sample}})$  in Eqs. (1) and (2) is estimated by the following equations for the linear least square fitting procedures, which is the same for  $A_{\text{H}_{2}\text{O}}$ .

$$\sigma(A_{\text{sample}}) = \frac{s_{y/x}(A_{\text{sample}})}{\left\{\sum_{i} \left(G_{i}^{2} - \overline{G_{i}^{2}}\right)^{2}\right\}^{0.5}},$$
(13)

where

$$s_{y/x}(A_{\text{sample}}) = \left\{ \frac{\sum_{j=1}^{2} \left[ \ln(I/I_0)_{\text{sample}} - \ln(I/\hat{I}_0)_{\text{sample}} \right]^2}{n-2} \right\}^{0.5}.$$
(14)

# 4.6. Calculation of the combined standard uncertainty of D

For both PFGSE and PFGSTE, the standard uncertainty of  $D_{\text{sample}}$  is calculated by

$$\sigma^{2}(D_{\text{sample}}) = \frac{\partial^{2}(D_{\text{sample}})}{\partial^{2}(A_{\text{sample}})} \sigma^{2}(A_{\text{sample}}) + \frac{\partial^{2}(D_{\text{sample}})}{\partial^{2}(D_{\text{H}_{2}\text{O},\text{ref}})} \sigma^{2}(D_{\text{H}_{2}\text{O},\text{ref}}) + \frac{\partial^{2}(D_{\text{sample}})}{\partial^{2}(\delta_{\text{H}_{2}\text{O}})} \sigma^{2}(\delta_{\text{H}_{2}\text{O}}) + \frac{\partial^{2}(D_{\text{sample}})}{\partial^{2}(A_{\text{H}_{2}\text{O}})} \sigma^{2}(A_{\text{H}_{2}\text{O}}) + \frac{\partial^{2}(D_{\text{sample}})}{\partial^{2}(A_{\text{H}_{2}\text{O}})} \sigma^{2}(A_{\text{H}_{2}\text{O}}) + \frac{\partial^{2}(D_{\text{sample}})}{\partial^{2}(\delta_{\text{sample}})} \sigma^{2}(\delta_{\text{sample}}) + \frac{\partial^{2}(D_{\text{sample}})}{\partial^{2}(A_{\text{sample}})} \sigma^{2}(A_{\text{sample}})$$
(15a)

$$\begin{aligned} \sigma^{2}(D_{\text{sample}}) &= \left(\frac{D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}^{2}(A_{\text{H}_{2}\text{O}} - \delta_{\text{H}_{2}\text{O}}/3)}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{sample}}) \\ &+ \left(\frac{A_{\text{sample}}\delta_{\text{H}_{2}\text{O}}^{2}(A_{\text{H}_{2}\text{O}} - \delta_{\text{H}_{2}\text{O}}/3)}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(D_{\text{H}_{2}\text{O},\text{ref}}) \\ &+ \left(\frac{2A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}(A_{\text{H}_{2}\text{O}} - \delta_{\text{H}_{2}\text{O}}/3)}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(\delta_{\text{H}_{2}\text{O}}) \\ &- \frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}^{2}}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(\delta_{\text{H}_{2}\text{O}}) \\ &+ \left(\frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}^{2}}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{H}_{2}\text{O}}) \\ &+ \left(\frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}^{2}}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{H}_{2}\text{O}}) \\ &+ \left(\frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}^{2}{\delta_{\text{sample}}^{2}(A_{\text{H}_{2}\text{O}} - \delta_{\text{H}_{2}\text{O}}/3)}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{H}_{2}\text{O}}) \\ &+ \left(-\frac{2A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}^{2}(A_{\text{H}_{2}\text{O} - \delta_{\text{H}_{2}\text{O}}/3)}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{H}_{2}\text{O}}) \\ &+ \left(-\frac{2A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}^{2}(A_{\text{H}_{2}\text{O} - \delta_{\text{H}_{2}\text{O}}/3)}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)\sigma^{2}(\delta_{\text{sample}}) \\ &+ \left(\frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}\delta_{\text{H}_{2}\text{O}}^{2}(A_{\text{H}_{2}\text{O} - \delta_{\text{H}_{2}\text{O}}/3)}{\delta_{\text{sample}}^{2}(A_{\text{sample}} - \delta_{\text{sample}}/3)^{2}A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{sample}). \end{aligned}{}$$

If one uses the same value of  $\Delta$  and  $\delta$  in both measurements of water and the sample, Eq. (15b) changes to Eq. (16) as follows:

$$\begin{aligned} \sigma^{2}(D_{\text{sample}}) &= \left(\frac{D_{\text{H}_{2}\text{O},\text{ref}}}{A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{sample}}) + \left(\frac{A_{\text{sample}}}{A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(D_{\text{H}_{2}\text{O},\text{ref}}) \\ &+ \left(\frac{2A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}}{\delta_{\text{sample}}A_{\text{H}_{2}\text{O}}} - \frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}}{3(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} (\delta_{\text{H}_{2}\text{O}}) \\ &+ \left(\frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}}{(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{H}_{2}\text{O}}) \\ &+ \left(\frac{D_{\text{H}_{2}\text{O},\text{ref}}A_{\text{sample}}}{\delta_{\text{sample}}A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(A_{\text{H}_{2}\text{O}}) \\ &+ \left(-\frac{2A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}}{\delta_{\text{sample}}A_{\text{H}_{2}\text{O}}} + \frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}}{3(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(\delta_{\text{sample}}) \\ &+ \left(\frac{A_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}}{\delta_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}} - \frac{\delta_{\text{sample}}D_{\text{H}_{2}\text{O},\text{ref}}}{3(A_{\text{sample}} - \delta_{\text{sample}}/3)A_{\text{H}_{2}\text{O}}}\right)^{2} \sigma^{2}(\Delta_{\text{sample}}) \end{aligned}$$

$$(16)$$

# 5. Results and discussion

Table 1 shows the results of self-diffusion coefficients and the associated uncertainties for the various solvents. Most of the uncertainties are within approximately 0.4%. Let us consider the contribution of different sources of uncertainty. Since Eq. (16) was used to estimate the uncertainties there are no contributions from  $\delta$  and  $\Delta$ , i.e., 0.000722% for the uncertainty of  $\delta$  and 0.00143% for the uncertainty of  $\Delta$ . Fig. 2 represents the uncertainty contribution of different sources to that of the self-diffusion coefficients of the measured solvents. Checking the size of the

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Table 1 Self-diffusion coefficients of the various solvents and the corresponding standard uncertainties (in bracket)

Solvent	$A_{\text{sample}} (\text{m}^2 \text{ s/T})$	$D_{\text{sample}} (\text{m}^2/\text{s})$
Water	-6.0392 (0.013042)	$2.299 \times 10^{-9} (0.398 \times 10^{-11})^{a}$
Benzene	-5.7537 (0.007034)	$2.219 \times 10^{-9} (0.663 \times 10^{-11})$
Methanol	-5.9494 (0.007146)	$2.295 \times 10^{-9} (0.683 \times 10^{-11})$
Ethanol	-2.7728(0.003482)	$1.070 \times 10^{-9} (0.321 \times 10^{-11})$
Chloroform	-6.0444(0.038788)	$2.332 \times 10^{-9} (1.61 \times 10^{-11})$
Cyclohexane	-3.7757 (0.011125)	$1.457 \times 10^{-9} \ (0.581 \times 10^{-11})$
Acetonitrile	-11.122 (0.025962)	$4.291 \times 10^{-9} (1.53 \times 10^{-11})$
Dichloromethane	-9.0243 (0.020827)	$3.482 \times 10^{-9} (1.24 \times 10^{-11})$
Hexane	-11.101 (0.027534)	$4.283 \times 10^{-9} (1.57 \times 10^{-11})$

<sup>a</sup> Given in [5] and see text.

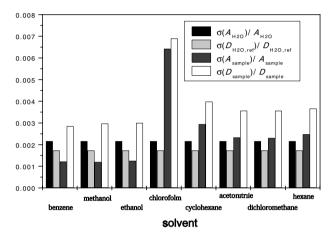


Fig. 2. Uncertainty contribution of the self-diffusion coefficients of the various solvents.

contribution, three main uncertainty sources equally affect the total standard uncertainty of  $D_{\text{sample}}$ , with the exception for chloroform. The large uncertainty of the diffusion coefficient of chloroform arises from the uncertainty of  $A_{\text{sample}}$ . This might come from a small radiation damping effect because chloroform has a higher density than the other solvents.

In contrast to this observation, Table 2 and Fig. 3 show different phenomena for a normal NMR tube. The uncertainties of the self-diffusion coefficients of the three solvents measured with a normal NMR tube are approximately five times larger than the values measured with the Shigemi NMR tube. In addition, the measured values are signifi-

Table 2 Self-diffusion coefficients and standard uncertainties for the normal 5 mm NMR tube

Solvent	$A_{\text{sample}} (\text{m}^2 \text{ s/T})$	$D_{\text{sample}} (\text{m}^2/\text{s})$
Water	-5.8043 (0.03966)	$2.299 \times 10^{-9} (0.398 \times 10^{-11})^{a}$
Benzene	-5.7626 (0.07331)	$2.3133 \times 10^{-9} (3.319 \times 10^{-11})$
Methanol	-6.4885(0.05000)	$2.605 \times 10^{-9} (2.684 \times 10^{-11})$
Chloroform	-5.5443 (0.1903)	$2.226 \times 10^{-9} (7.697 \times 10^{-11})$

<sup>a</sup> Given in [5] and see text.

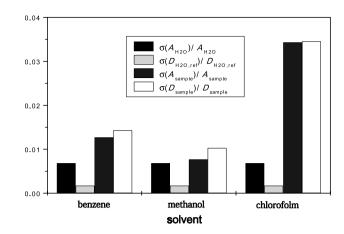


Fig. 3. Uncertainty contribution to the self-diffusion coefficients of the various solvents using a normal 5 mm NMR tube.

cantly different from those taken with the Shigemi NMR tube. For methanol, the difference between the measured self-diffusion coefficient of the Shigemi NMR tube and that with the normal tube is almost 13.5%. On the other hand, the differences of benzene and chloroform are about 4.5%. These findings could correlate with the order of the Prandtl numbers, factors relating to the Rayleigh-Bénard convection, i.e., 81.7 for benzene, 28.3 for methanol, and 71.9 for chloroform [55]. This result could be caused by the effect of the convection of the solvent. The existence of convection effects was confirmed by measuring the diffusion coefficient with increasing  $\Delta$  [51,56]. When we measured the self-diffusion coefficients of benzene for  $\Delta$  set to 50 and 100 ms, the observed difference for those two  $\Delta$  was almost 0.4% when using the Shigemi NMR tube. Using the normal NMR tube, the observed difference of the diffusion coefficients of benzene is 16.4% between  $\Delta = 50 \text{ ms}$ and  $\Delta = 100$  ms. This finding indicates that convection can significantly affect the measurement of self-diffusion coefficients. The largest uncertainty contribution to the self-diffusion coefficient is caused by the uncertainty of

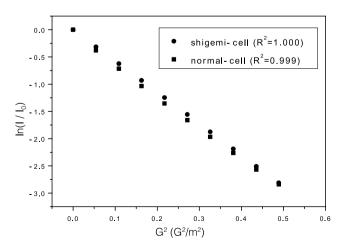


Fig. 4. PFG-NMR attenuation plots for benzene measured with Shigemiand normal-NMR tubes.

Table 3 Self-diffusion coefficients of the various solvents  $(D \times 10^{-9} \text{ m}^2/\text{s})^a$ 

Solvent This work		Other reference value (diaphragm, capillary-cell)	Other reference value (NMR)	
Water	Calibration	2.2991 [21]		
Benzene	2.219	2.15[22], 2.13[23], 2.21[24], 2.231[25], 2.18[27], 2.27[28], 2.207[29]	2.26[40], 2.21[42], 2.1[45]	
MeOH	2.295	2.34[24], 2.21[26], 2.37[27], 2.32[30], 2.27[31], 2.425[32]	3.50[41], 2.41[42], 2.415[43], 2.3[45]	
EtOH	1.070	1.05[22], 1.02[24,27], 1.01[30]	1.08[42], 1.07[43], 1.0[45], 1.085[47]	
CHCl3	2.332	2.42[33]	2.83[41]	
Cyclohexane	1.457	1.475[34], 1.43[35]	1.47[40], 1.42[43]	
Acetonitrile	4.291	4.34[36], 5.4[37]	4.37[42], 4.39[43]	
Dichloromethane	3.482		4.00[44]	
Hexane	4.283	4.13[38], 4.263[39]	4.25[43], 4.2[45], 4.21[46]	

<sup>a</sup> References in brackets.

 $A_{\text{sample}}$  as given in Fig. 3, where data was measured with the normal NMR tube, are presented. In contrast to Fig. 1, the contribution of the uncertainty of  $D_{\text{H}_2\text{O},\text{ref}}$  is low. This means the uncertainty of *t*, the diffusion coefficient measured using the normal NMR tube, is quite large because of the convection effects. Thus, the observed diffusion coefficients are not reliable. Fig. 4 clearly illustrates the cause of the difference in the uncertainty of  $A_{\text{sample}}$  between the Shigemi tube and the normal tube for benzene. The data measured with the latter tube exhibits a concave curve in contrast to that measured using the former tube.

The self-diffusion coefficients of the solvent molecules together with the estimated values of the earlier works are summarized in Table 3. However, one can say that choosing the primary reference value is quite important for PFG-NMR measurements. In such a case, what must be done is to use the literature value of the diffusion coefficient of water or benzene. The former value was obtained from mass extrapolation, but was examined so carefully [21,48] that one can assume that the result is reliable. The latter has small mass effects and therefore, the tracer method should show a small deviation [57]. Although there are many measurements of the diffusion coefficient of benzene from [22–30], we measured the self-diffusion coefficient of benzene using water as a reference. Our result showed that the Mills et al. data [29] is the most reliable since the value the authors report is within the standard uncertainty. Our observed values are systematically approximately 2% lower than the Holz et al. data [43], with the exception of methanol (5%). However, these values are comparable to each other and useful since the data is of cyclohexane, acetonitrile, dichloromethane, and hexane; the most available solvents in photochemistry and there are not much utilized data of for those solvents.

The background gradients effect problem affects the determination of the diffusion coefficient [58] caused by short height of sample, however the difference of the diffusion coefficients at the measurements for a different sample thickness from 2 to 1 mm is less than 0.04%. The estimation of the contribution of this uncertainty is quite complicated, since this contains not only uncertainty of background gradients effect but also those of convection

and RF inhomogeneity artifact effects. Therefore, the detailed discussion on this effect has not done in this work.

# 6. Conclusions

The determination of diffusion coefficients using the PFG-NMR method for the molecules of common solvents, has been outlined in this work. The five main uncertainty sources  $A_{\rm H_2O}$ ,  $A_{\rm sample}$ ,  $D_{\rm H_2O,ref}$ ,  $\delta$ , and  $\Delta$ ) were chosen in order to estimate the total standard uncertainty of  $D_{\text{sample}}$ . In this paper we outlined the measurement procedure and the estimation of the uncertainties of the self-diffusion coefficient for methanol, ethanol, benzene, dichloromethane, chloroform, hexane, cvclohexane, and acetonitrile. Using the Shigemi NMR tubes, we successfully performed accurate measurements of the known solvents, with an accurate evaluation of the uncertainty; where the total standard uncertainties were found to be approximately 0.4%, with the exception of chloroform. In these measurements, the three uncertain sources  $A_{\rm H_2O}$ ,  $A_{\rm sample}$ , and  $D_{\rm H_2O,ref}$  equally affect the total standard uncertainty of  $D_{\text{sample}}$  in the measurement of the self-diffusion coefficient. The uncertainties in the self-diffusion coefficients measured with a normal NMR tube are larger than those of the Shigemi NMR tube because of the larger uncertainty of  $A_{H_2O}$  and  $A_{sample}$  in the normal NMR tube. This result might arise from convection effects due to the assumption of Rayleigh-Bénard convection. It is suggested that the refined measurement procedure employed in this study to measure the diffusion coefficients, using the PFG-NMR method, is a significant improvement for the precise evaluation of transport and chemical reaction phenomena in a solution in both the experimental and computational fields of study.

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